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Collision Analysis of mIoT network with Power Ramping Scheme

Nan Jiang, Yansha Deng, Arumugam Nallanathan, Xin Kang, and Tony Q. S. Quek

Abstract—The Random Access (RA) procedure is used to request channel resources for the uplink data transmission in the cellular-based massive Internet of Things (mIoT). To ease the RA failure and the network congestion, power ramping (PR) technique is used to step up the preamble transmit power after each unsuccessful RA attempt. In this paper, we develop a traffic-aware spatio-temporal model to analyze the PR scheme in the mIoT network, where the Signal-to-Interference-and-Noise Ratio (SINR) outage and collision events jointly determine the traffic evolution and the RA success probability. Compared with existing literature only modelled collision from single cell perspective, we model both the SINR outage and the collision from the network perspective. Based on this analytical model, we derive the exact expression for the RA success probability to show the effectiveness of the PR scheme. Our results show that the geometry PR scheme with smooth increased transmission power is effective in heavy traffic scenario in terms of increasing the RA success probability.

I. INTRODUCTION

To support rapid proliferation of innovative applications, massive Internet of Things (mIoT) has gained unprecedented momentum, which aims at providing reliable wireless access for massive IoT devices and diversification of data traffic [1]. Cellular-based technologies were considered as potential solutions, however providing reliable and efficient access mechanisms for massive IoT devices at the same time is still a key challenge [2–4].

In the cellular-based mIoT network, contention-based Random Access (RA) procedure is considered as the main technology for uplink channel resources requesting. In more detail, an IoT device randomly selects a non-dedicated preamble (i.e., orthogonal pseudo code, such as Zadoff-Chu sequence) transmitting to its associated base station (BS) in the 1st step of RA [5]. As single preamble provides single RA opportunity, preambles contention among IoT devices represents their competition of uplink channel resources. When competing simultaneously, IoT devices choosing the same preamble bring mutual interference and collision risks in preamble detection, resulting in performance degradation in terms of high RA failure probability of mIoT communications [2, 6].

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A collision occurs at Step 1 of RA when a BS successfully decodes two or more same preambles from different IoT devices simultaneously, such that the BS cannot serve any colliding IoT devices, and these IoT devices need to restart the RA procedure in the next available RA time slot. The 3GPP has investigated a number of mIoT reports regarding the limits of the contention-based RA [4, 7], which point out that the efficient RA schemes are essential for improving the success RA performance under limited channel resources.

Power Ramping (PR) scheme is an efficient RA scheme, which is commonly used in the LTE network [5]. This scheme is deemed as a potential solution to improve the success RA performance in the mIoT network, due to that it is easy to implement with less modification of the standard at the medium access control (MAC) layer [7, 8]. In [9], fixed, linear, and geometric power ramping schemes are compared. In [8], the authors evaluate the impact of the PR scheme, retransmission attempts and Physical Downlink Control Channel (PDCCH) deficiency in LTE/LTE-A networks. However, due to the analytical difficulty in capturing both interference and collision, most works about the PR scheme was studied from the single cell point of view.

In this paper, we model the SINR outage and the collision to analyze the success RA from the network point of view. The probability that the received SINR at a randomly chosen BS exceeds a certain threshold γ_{th} has been studied in many stochastic geometry works [10, 11]. To the best of our knowledge, there has been no work in the literature considered and analyzed collision problem during RA via stochastic geometry so far. We derive the general exact expression for the RA success probability in each time slot with infinite number of power level units, where the queue evolution is analyzed using probability theory based on our previous work [12]. Finally, we verify the RA success probability of the PR scheme using our proposed realistic simulation framework, which captures the randomness location, preamble transmission, RA collision, as well as the real packets arrival, accumulation, and departure of each IoT device in each time slot. The numerical results show that the geometry PR scheme with smooth increased transmission power is effective in heavy traffic scenario in terms of increasing the RA success probability.

The rest of the paper is organized as follows. Section II introduces the system model. Sections III presents the analytical results for the RA success probabilities in each time slot with the PR scheme. Section IV provide numerical results. Finally, Section V concludes the paper.

II. SYSTEM MODEL

We consider a traffic-aware spatio-temporal model for the cellular-based mIoT network: 1) the spatial model of BSs and IoT devices are distributed in \mathbb{R}^2 following two independent homogeneous Poisson point process (PPP), Φ_B and Φ_D , with intensities λ_B and λ_D , respectively; 2) the temporal model captures the packets arrival and departure at each IoT device in each time slot. We intend to analyzing the contention-based RA in the mIoT network, and assume that the actual intended packet transmission is always successful if the corresponding RA succeeds. Note that the data transmission after a successful RA can be easily extended following the analysis of preamble transmission probability in RA. Here, we limit ourselves to focus on the impact of massive access to RA procedure.

In RA, an IoT device randomly selects a preamble from available preamble pool for transmitting to its associated BS via Physical Random Access CHannel (PRACH) in the step 1 [5, 12]. We assume ξ denotes the number of available preambles in the mIoT network. Without loss of generality, each IoT device has an equal probability ($1/\xi$) to choose a specific preamble, and the average density of IoT devices using same preamble is $\lambda_{Dp} = \lambda_D/\xi$, where λ_{Dp} is measured with unit devices/preamble/km². We focus on analyzing preamble contention in the mIoT network, and assume that the step 2, 3, and 4 (i.e., control information exchange via normal up-link/downlink channels) of RA are always successful whenever the step 1 is successful.

As mentioned earlier, we consider two independent link-outage conditions: 1) the BS cannot decode the preamble due to the low received SINR; 2) the BS successfully decodes the same preamble from two or more IoT devices in the same time, such that the collision occurs. According to [3, 4, 13], we assume collision events are detected by BS after it decodes the preambles in the step 1 of RA, and then no response will be feedback from the BS to the IoT devices, such that it can not proceed to the next step of RA.

Different from [12, 14, 15], where the locations of all IoT devices are fixed all the time, we assume the location of each active IoT device choosing same preamble varies in different time slot due to that: 1) the IoT devices are moving; and 2) the IoT devices randomly choose a preamble in each RA attempt, such that the set of active IoT devices using the same preamble changes in different time slot.

A. Physical Layer Description

We assume each IoT device associates to its geographically nearest BS, and a standard power-law path-loss model is considered, where the path-loss is inversely proportional to distance r with the path-loss exponent α [10, 14–18]. The identically distributed (i.i.d.) Rayleigh fading channel is considered, where the power gains h is assumed to be exponentially distributed random variables with unit mean. Similar as [10, 12], we apply a full path-loss inversion power control, where each IoT device controls its transmit power by compensating for its own path-loss to maintain the average received signal power in the BS equalling to a same threshold

ρ . We also assume the density of BSs is high enough and no IoT device suffers from truncation outage [12].

B. MAC Layer Description

We consider a time-slotted cellular-based mIoT network, where the PRACH with duration τ_c are reserved in the uplink channel and repeated in the system with a certain period that specified by the BS (i.e., a gap interval between two neighbor PRACH is τ_g) [5]. We assume a geometric new packets arrival process in each time slot at each IoT device, which is modelled as independent Poisson arrival process $N_{\text{New}}^m \sim \text{Pois}(\mu_{\text{New}}^m)$, with intensity $\mu_{\text{New}}^m = (\tau_c + \tau_g)\varepsilon_{\text{New}}^m$ [12, 19, 20]. More details about RA structure and traffic model (i.e., packets arriving and leaving) can be found in our previous work [12, Section II.C].

We assume each IoT device has an infinite buffer to store queueing packets until their successful transmission, where none of packets will be dropped off, and each IoT device transmits packets via a First Come First Serve packets scheduling scheme [21]. The queue status of each IoT device is evolved depending on transmission condition over time, which has been detailed and analyzed in our previous work [12, Section II.C and IV.A]. Briefly speaking, a packet is removed from the buffer once it has been successfully transmitted (step 1 of RA of that IoT device is successful), otherwise, it will wait in the first place of the queue, and this IoT device will reattempt to access the network in the next available RA.

In RA, the power ramping technique is used to favor the delayed preambles by stepping up the transmission power after each unsuccessful RA attempt. In doing so, the IoT device uses the full path-loss inversion power control to maintain the average received preamble power at a higher power level in the next RA attempt, where κ_i denotes the power level unit in the i th RA attempt by adjusting the target received preamble power at the BS equal to $\kappa_i\rho$ [9] (i.e., $\kappa_1 < \kappa_2 < \dots < \kappa_i < \dots < \kappa_J$). Note that κ_J is the maximum allowable power level unit.

C. SINR Expression

Different preambles represent orthogonal sub-channels, such that only IoT devices choosing the same preamble have correlations. Based on Slivnyak's theorem [22], we formulate the SINR transmitted from a typical IoT device using the power level unit κ_l ($l \in [1, J]$) as

$$\text{SINR}_l = \frac{\kappa_l \rho h_o}{\sum_{j=1}^J \left(\sum_{u_k^j \in \mathcal{Z}_j} \kappa_j \rho h_k^j + \sum_{u_i^j \in \mathcal{X}_j} P_i^j h_i^j \left\| u_i^j \right\|^{-\alpha} \right) + \sigma^2}, \quad (1)$$

where ρ is the full path-loss inversion power control threshold, h_o is the channel power gain from the typical IoT device to its associated BS, \mathcal{Z}_j is the set of active intra-cell interfering IoT devices transmitting with the power level unit κ_j , \mathcal{X}_j is the set of active inter-cell interfering IoT devices transmitting with the power level unit κ_j , $\|\cdot\|$ is the Euclidean norm, u_i^j is the distance between the i th inter-cell IoT device and the typical BS, $P_i^j = \kappa_j \rho (r_i^j)^\alpha$ is the actual transmit power of the

i th inter-cell IoT device with the distance from its associated BS r_i^j , and σ^2 is the noise power.

III. RA SUCCESS PROBABILITY IN THE SINGLE TIME SLOT MODEL

In this section, we provide a general single time slot analytical model. Note that in the 1st time slot, the queue status (number of packets in buffer) of each IoT device only depends on the new packets arrival process $\text{Pois}(\mu_{\text{New}}^1)$, and all the IoT devices transmit the preamble without power ramping (i.e., $\kappa_1 = 1$). We perform the analysis on a BS associating with a randomly chosen active IoT device in terms of the RA success probability \mathcal{P}^1 that is defined as

$$\mathcal{P}^1 = \sum_{n_1=0}^{\infty} \left\{ \underbrace{\mathbb{P}[N_1 = n_1]}_{\text{I}} \underbrace{\mathbb{P}[\text{SINR}_o \geq \gamma_{\text{th}} | N_1 = n_1]}_{\text{II}} \underbrace{\left(\prod_{i=1}^{n_1} \mathbb{P}[\text{SINR}_i < \gamma_{\text{th}} | N_1 = n_1] \right)}_{\text{III}} \right\}, \quad (2)$$

where γ_{th} is the SINR threshold, N_1 is the number of intra-cell interfering IoT devices (i.e., transmitting the same preamble as the typical IoT device simultaneously), SINR_o and SINR_i are the received SINR of preamble from the typical and the i th interfering IoT device following from (1). In (2), I is the probability of n_1 number of interfering IoT devices located in the typical BS, II and III represent the probability that the typical IoT device successfully transmits a preamble and the probability that the other n_1 intra-cell interfering IoT devices fail to transmit a preamble, respectively. Next, we derive the Probability Mass Function (PMF) of the number of interfering IoT devices n_1 is represented in the following Lemma.

Lemma 1. *The PMF of the number of interfering IoT devices n_1 in a Voronoi cell is obtained as [12, Eq.(12)]*

$$\mathbb{P}[N_1 = n_1] = \frac{c^{(c+1)} \Gamma(n_1 + c + 1) \left(\frac{\mathcal{T}^1 \lambda_{Dp}}{\lambda_B} \right)^{n_1}}{\Gamma(c + 1) \Gamma(n_1 + 1) \left(\frac{\mathcal{T}^1 \lambda_{Dp}}{\lambda_B} + c \right)^{n_1 + c + 1}}, \quad (3)$$

where $c = 3.575$ is a constant, $\Gamma(\cdot)$ is the gamma function, λ_{Dp} is the density of IoT devices using the same preamble, and $\mathcal{T}^1 = \mathbb{P}\{N_{\text{New}}^1 > 0\} = 1 - e^{-\mu_{\text{New}}^1}$ is the active probability of each IoT device in the 1st time slot.

Then, we derive the RA success probability in the 1st time slot \mathcal{P}^1 in the following theorem.

Theorem 1. *In the depicted cellular-based mIoT network, the RA success probability of a randomly chosen IoT device in the 1st time slot is derived as*

$$\mathcal{P}^1 = \sum_{n_1=0}^{\infty} \Omega(n_1) \Theta(n_1) (1 - \Theta(n_1))^{n_1}, \quad (4)$$

where $\Omega(n_1)$ is given in (3), and $\Theta(n_1)$ is the preamble transmission success probability that the received SINR at the BS from a randomly chosen IoT device exceeds a certain

threshold γ_{th} conditioning on a given number of interfering IoT devices in that cell n_1 is expressed as [12, Eq.(14)]

$$\begin{aligned} \Theta(n_1) &= \mathbb{P}\left[\frac{\kappa_1 \rho h_o}{\mathcal{I}_{\text{intra}} + \mathcal{I}_{\text{inter}} + \sigma^2} \geq \gamma_{\text{th}} \middle| N_1 = n_1\right] \\ &= \exp\left(\frac{\gamma_{\text{th}}}{\kappa_1 \rho} \sigma^2\right) \mathcal{L}_{\mathcal{I}_{\text{intra}}}\left(\frac{\gamma_{\text{th}}}{\kappa_1 \rho} \middle| N_1 = n_1\right) \mathcal{L}_{\mathcal{I}_{\text{inter}}}\left(\frac{\gamma_{\text{th}}}{\kappa_1 \rho}\right) \\ &= \frac{\exp\left(-\frac{\gamma_{\text{th}} \sigma^2}{\rho} - 2(\gamma_{\text{th}})^{\frac{2}{\alpha}} \frac{\mathcal{T}^1 \lambda_{Dp}}{\lambda_B} \int_{(\gamma_{\text{th}})^{\frac{1}{\alpha}}}^{\infty} \frac{y}{1+y^{\alpha}} dy\right)}{(1 + \gamma_{\text{th}})^{n_1}}, \end{aligned} \quad (5)$$

where $\kappa_1 = 1$, $\mathcal{L}_{\mathcal{I}_{\text{intra}}}(\cdot)$ and $\mathcal{L}_{\mathcal{I}_{\text{inter}}}(\cdot)$ denote the Laplace Transforms of the aggregate intra-cell interference $\mathcal{I}_{\text{intra}}$ and the aggregate inter-cell interference $\mathcal{I}_{\text{inter}}$, respectively.

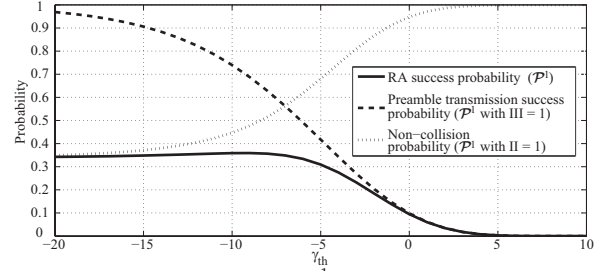


Fig. 1: Comparing RA success probability (\mathcal{P}^1), preamble transmission success probability (\mathcal{P}^1 with III = 1), and non-collision probability (\mathcal{P}^1 with II = 1). The parameters are $\lambda_B = 10$ BS/km², $\lambda_{Dp} = 100$ IoT devices/preamble/km², $\rho = -90$ dBm, $\sigma^2 = -90$ dBm, and $\mu_{\text{New}}^1 = 0.1$ packets/time slot.

In (4), it can be shown that the preamble transmission success probability of the typical IoT device is inversely proportional to the received SINR threshold γ_{th} , and the preamble transmission failure probabilities of other interfering IoT devices are directly proportional to the received SINR threshold γ_{th} , which leads to the fact that the non-collision probability (i.e., the probability of a successful transmission preamble does not collide with others) of the typical IoT devices is also directly proportional to the received SINR threshold γ_{th} . Therefore, a tradeoff between preamble transmission success probability and non-collision probability is observed. For illustration, the relationship among RA success probability, the preamble transmission success probability, and the non-collision probability are shown in Fig. 1.

IV. RA SUCCESS PROBABILITY WITH THE POWER RAMPING SCHEME

In this section, we analyze the RA success probability of the cellular-based mIoT network in each time slot with the PR scheme. Due to the PR scheme, IoT devices can transmit preamble using different power level units in the system depending on their current preamble transmission attempts. Consequently, IoT devices using different power level units are correlated (i.e., the current j th power level unit is caused by the previous $j - 1$ failure RA attempts), which greatly complicate the performance analysis. According to the thinning theory, the IoT devices using each power level unit constitute a PPP and these PPPs are correlated. Therefore, to calculate the RA success probability, the main challenge is evaluating the distribution of the active IoT devices transmitting with different power level units. To ease the derivation of collision event and the intra-cell interference, we first focus on deriving

$$\mathbb{P}[N_j = n_j | N_1 = n_1, \dots, N_{j-1} = n_{j-1}] = \frac{(\mathcal{T}_{\kappa_j} \lambda_{Dp})^{n_j} \left(\left(\sum_{i=1}^{j-1} \mathcal{T}_{\kappa_i} \right) \lambda_{Dp} + c \lambda_B \right)^{\left(\sum_{i=1}^{j-1} n_i \right) + c + 1} \Gamma \left(\left(\sum_{i=1}^j n_i \right) + c + 1 \right)}{\Gamma(n_j + 1) \Gamma \left(\left(\sum_{i=1}^{j-1} n_i \right) + c + 1 \right) \left(\left(\sum_{i=1}^j \mathcal{T}_{\kappa_i} \right) \lambda_{Dp} + \lambda_B c \right)^{\left(\sum_{i=1}^j n_i \right) + c + 1}}, \quad (6)$$

$$\mathcal{P}_{\kappa_l}^m = \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \dots \sum_{n_J=0}^{\infty} \left\{ \mathbb{P}[N_l = n_l] \left(\prod_{j=1, j \neq l}^J \mathbb{P}[N_j = n_j | N_l = n_l, N_1 = n_1, \dots, N_{j-1} = n_{j-1}] \right) \right. \\ \left. \left(\mathbb{P} \left[\frac{\kappa_l \rho h_o}{\sum_{i=1}^J (\mathcal{I}_{\text{inter}_i}^m + \mathcal{I}_{\text{intra}_i}^m) + \sigma^2} \geq \gamma_{\text{th}} \right] \prod_{j=1}^J \mathbb{P} \left[\frac{\kappa_j \rho h_o}{\sum_{i=1}^J (\mathcal{I}_{\text{inter}_i}^m + \mathcal{I}_{\text{intra}_i}^m) + \sigma^2} < \gamma_{\text{th}} \right]^{n_j} \middle| N_1 = n_1, \dots, N_J = n_J \right) \right\}. \quad (7)$$

the PMF of the number of interfering IoT devices transmitting with each power level unit in a specific cell.

A. PMF of the Number of Interfering IoT Devices

We denote the j th power level unit as κ_j ($j \in [1, J]$), and the number of interfering IoT devices transmitting same preamble with κ_j being located in the typical Voronoi cell as N_j . The active probability of IoT devices transmitting with the power level units κ_j is denoted as \mathcal{T}_{κ_j} . Note that the active probabilities with different power level units are derived based on iteration process, which will be represented in (14). The PMF of N_1 has been given in (3). Then, we derive the PMF of N_j ($j = 2, 3, 4, \dots, J$) conditioning on the known number of IoT devices with other power levels ($N_1=n_1, N_2=n_2, \dots, N_{j-1}=n_{j-1}$) in the following theorem.

Lemma 2. *The PMF of N_j number of IoT devices transmitting with the power level unit κ_j in a Voronoi cell conditioning on number of IoT devices with other power levels ($N_1=n_1, N_2=n_2, \dots, N_{j-1}=n_{j-1}$) and the typical IoT device transmitting with the power level unit κ_1 is given in (6).*

Proof. See Appendix A. \square

B. RA Success Probability

In the PR scheme, we assume the maximum allowable power level unit is κ_J . The RA success probability of the IoT device transmits preamble with the l th power level unit κ_l in the m th time slot $\mathcal{P}_{\kappa_l}^m$ is written as (7). In (7), $\mathcal{I}_{\text{inter}_i}^m$ and $\mathcal{I}_{\text{intra}_i}^m$ denote the aggregate inter-cell and intra-cell interference generating by IoT devices transmitting with the i th level power unit κ_i , respectively. Next, we present the RA success probability of a randomly chosen IoT device with multiple levels PR scheme (i.e., the maximum allowable power level unit is κ_J ($J \geq 2$)) in the m th time slot in the next theorem.

Theorem 2. *The RA success probability of a randomly chosen IoT device (i.e. each active IoT device transmitting preamble with any power level unit is fairly chosen) in the m th time slot is derived as*

$$\mathcal{P}_{\text{all}}^m = \left(\sum_{i=1}^J \mathcal{T}_{\kappa_i}^m \mathcal{P}_{\kappa_i}^m \right) / \mathcal{T}_{\text{all}}^m, \quad (8)$$

where J is the maximum allowable power level, the RA success probability of IoT devices transmitting with the power level unit κ_l ($l \in [1, J]$) in the m th time slot is derived as

$$\mathcal{P}_{\kappa_l}^m = \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \dots \sum_{n_J=0}^{\infty} \left\{ \Theta(m, l, l, \vec{n}) \prod_{j=1}^J \Omega(m, l, j, \vec{n}) \right. \\ \left. \left(1 - \Theta(m, l, j, \vec{n}) \right)^{n_j} \right\}. \quad (9)$$

In (9), $\vec{n} = \{n_1, \dots, n_J\}$, the probability that the number of interfering IoT devices transmitting with the power level unit κ_l $\Omega(m, l, l, \vec{n})$ is given (3), the probability that the number of IoT devices transmitting with the power level unit κ_j (when $j \neq l$) $\Omega(m, l, j, \vec{n})$ is given in (6), and the preamble transmission success probability that the received SINR from an IoT device transmitting with the power level unit κ_l exceeds the certain threshold γ_{th} is derived as

$$\Theta(m, l, j, \vec{n}) = \exp \left(- \frac{\gamma_{\text{th}} \sigma^2}{\kappa_j \rho} - \frac{2 \lambda_{Dp} (\gamma_{\text{th}})^{\frac{2}{\alpha}}}{\lambda_B} \times \right. \\ \left. \sum_{i=1}^J \left(\frac{\kappa_i}{\kappa_j} \right)^{\frac{2}{\alpha}} \mathcal{T}_{\kappa_i}^m \int_{(\gamma_{\text{th}} \frac{\kappa_i}{\kappa_j})^{\frac{1}{\alpha}}}^{\infty} \frac{y}{1 + y^{\alpha}} dy \right) / \Xi(m, l, j, \vec{n}), \quad (10)$$

where

$$\Xi(m, l, j, \vec{n}) = \begin{cases} \prod_{i=1}^J (1 + \gamma_{\text{th}} \frac{\kappa_i}{\kappa_j})^{n_i}, & j = l \\ (1 + \gamma_{\text{th}} \frac{\kappa_l}{\kappa_j})^{n_l+1} (1 + \gamma_{\text{th}})^{n_j-1} \prod_{i=1, i \neq l, j}^J (1 + \gamma_{\text{th}} \frac{\kappa_i}{\kappa_j})^{n_i}, & j \neq l \end{cases} \quad (11)$$

Note that $\mathcal{T}_{\kappa_i}^m$ is derived based on iteration process, which will be given in (14).

Proof. See Appendix B. \square

The RA success probabilities are derived based on the iteration process. We assume m is a variable that denotes the time slot from 2 to M . The iteration process for calculating the

RA success probability in the M th time slot $\mathcal{P}_{\text{all}}^M$ is shown in Fig. 2. Details of this process are described by the following:

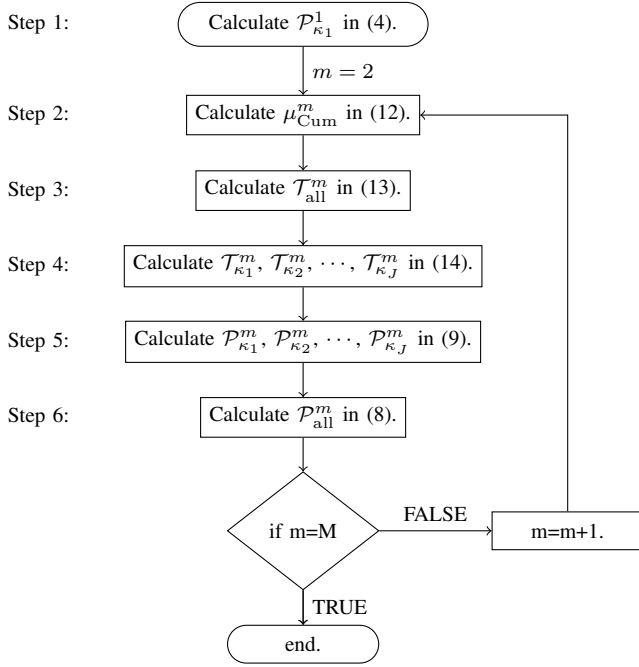


Fig. 2: Flowchart for deriving the RA success probability in the M th time slot with the PR scheme $\mathcal{P}_{\text{all}}^M$.

- Step 1: Calculate the RA success probability in the 1st time slot $\mathcal{P}_{\kappa_1}^1$ in (4);
- Step 2: Calculate the intensity of accumulated packets μ_{Cum}^m in the m th time slot via Poisson approximation queue status analysis approach, which is given in our previous work [12, Section IV.A]. The intensity of number of accumulated packets in the m th time slot μ_{Cum}^m is

$$\mu_{\text{Cum}}^m = \mu_{\text{New}}^{m-1} + \mu_{\text{Cum}}^{m-1} - \sum_{i=1}^J \mathcal{T}_{\kappa_i}^{m-1} \mathcal{P}_{\kappa_i}^{m-1}; \quad (12)$$

- Step 3: Calculate the active probability of each IoT device in the m th time slot $\mathcal{T}_{\text{all}}^m$ using

$$\mathcal{T}_{\text{all}}^m = 1 - e^{-\mu_{\text{New}}^m - \mu_{\text{Cum}}^m}; \quad (13)$$

- Step 4: Calculate the active probability of each IoT device transmitting with the power level unit κ_i ($i \in (1, J)$) in the m th time slot $\mathcal{T}_{\kappa_i}^m$ using

$$\mathcal{T}_{\kappa_i}^m = \begin{cases} \mathcal{T}_{\text{all}}^m - \sum_{i=1}^J \mathcal{T}_{\kappa_i}^{m-1} (1 - \mathcal{P}_{\kappa_i}^{m-1}), & i = 1 \\ (1 - \mathcal{P}_{\kappa_{i-1}}^{m-1}) \mathcal{T}_{\kappa_{i-1}}^{m-1}, & i \neq 1, i \neq J \\ (1 - \mathcal{P}_{\kappa_{i-1}}^{m-1}) \mathcal{T}_{\kappa_{i-1}}^{m-1} + (1 - \mathcal{P}_{\kappa_i}^{m-1}) \mathcal{T}_{\kappa_i}^{m-1}, & i = J \end{cases} \quad (14)$$

where $\mathcal{P}_{\kappa_i}^{m-1}$ is the RA success probability of the IoT device transmitting with the power level unit κ_i in the $(m-1)$ th time slot given in (9);

- Step 5: Calculate the RA success probabilities of IoT devices transmitting with power level unit κ_l ($l = 1, 2, \dots, J$) in the m th time slot $\mathcal{P}_{\kappa_l}^m$ using (9);

- Step 6: Calculate the RA success probability $\mathcal{P}_{\text{all}}^m$ using (8).

Repeating the step 2 to 6 until $m = M$, the RA success probability in the M th time slot $\mathcal{P}_{\text{all}}^M$ is obtained.

V. NUMERICAL RESULTS

In this section, we validate the derived analytical results via independent system level simulations. The BSs and IoT devices are deployed via independent PPPs in a 400 km² area. Note that we simulate the real buffer at each IoT device to capture the packets accumulated process evolved over time. In all figures of this section, “Analysis” and “Simulation” are abbreviated as “Ana.” and “Sim.”, respectively. Unless otherwise stated, we choose the same new packets arrival rate for each time slot ($\mu_{\text{New}}^1 = \mu_{\text{New}}^2 = \dots = \mu_{\text{New}}^m = 0.1$ packets/time slot), $\sigma^2 = -90$ dBm, $\rho = -90$ dBm, $\gamma_{\text{th}} = 0$ dB, $\alpha = 4$, $\lambda_B = 10$ BS/km². Unless otherwise stated, we consider the power level unit $\kappa_1 = 1$ as well as the maximum allowable power level unit $\kappa_J = \kappa_2 = 10$ for the PR scheme.

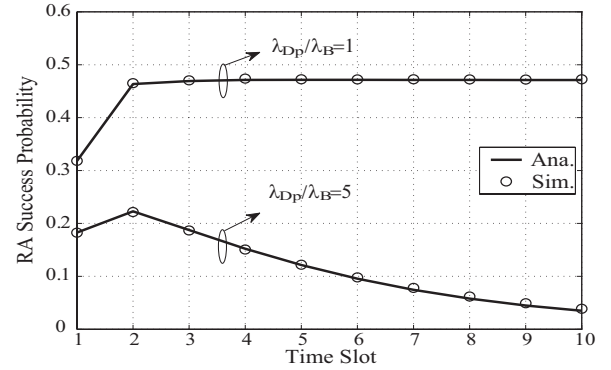


Fig. 3: RA success probability in each time slot with the PR scheme.

Fig. 3 plots the RA success probability of a randomly chosen IoT device within the 10 time slots. We consider a light traffic LTE network scenario ($\lambda_{DP}/\lambda_B = 1$) in Fig. 3(a) and a severer traffic mIoT network scenario ($\lambda_{DP}/\lambda_B = 5$) in Fig. 3(b), respectively. The analytical curves of the PR scheme $\mathcal{P}_{\text{all}}^m$ is plotted using (8). The close match between the analytical curves and simulation points validate the accuracy of developed spatio-temporal mathematical framework. It is observed that the RA success probabilities in Fig. 3(a) outperform that in Fig. 3(b), where the reasons can be concluded as: 1) Increasing the density of IoT devices increases the aggregate interference, which degrades the received SINR at the associated BS; 2) Increasing the density of IoT devices increases the number of IoT devices using the same preamble, which leads to increased collision probability.

Fig. 4 plots the RA success probabilities with the PR scheme at the 10th time slot $\mathcal{P}_{\text{all}}^{10}$ versus the density ratio between IoT devices transmitting the same preamble and BSs λ_{DP}/λ_B . We study the geometric PR scheme, where the transmit power steps up following the policy $\kappa_l = g^{l-1}$ (i.e., g is a constant denoting the root of power increase, l is the current power level, and $l \leq J$, where J is the maximum power level), and its effectiveness has been shown in [9]. Comparing the PR schemes with $J = 2$, the RA success probabilities follow

$\mathcal{P}_{\text{all}}^{10}(J = 2, g = 8) > \mathcal{P}_{\text{all}}^{10}(J = 2, g = 4) > \mathcal{P}_{\text{all}}^{10}(J = 2, g = 2)$, due to that increasing g results in higher received SINR of reattempt access and lower collision probability. We also notice that $\mathcal{P}_{\text{all}}^{10}(J = 5, g = 2)$ performs worse than $\mathcal{P}_{\text{all}}^{10}(J = 2, g = 8)$ before a certain density ratio, due to that in the low density ratio region, the network condition prefers large power gap, as this is effective in improving the received SINR of reattempt access and reducing the collision probability (i.e., most packets only suffer from little times of RA fails leading to that IoT devices always use small power level unit to transmit preambles). After that density ratio, $\mathcal{P}_{\text{all}}^{10}(J = 5, g = 2)$ surpasses $\mathcal{P}_{\text{all}}^{10}(J = 2, g = 8)$, due to that in the high density ratio region, the case with $J = 5$ and $g = 2$ ($\kappa_1, \dots, \kappa_5 = 1, 2, 4, 8, 16$) has relatively smooth increase in power that decreases the high aggregate interference.

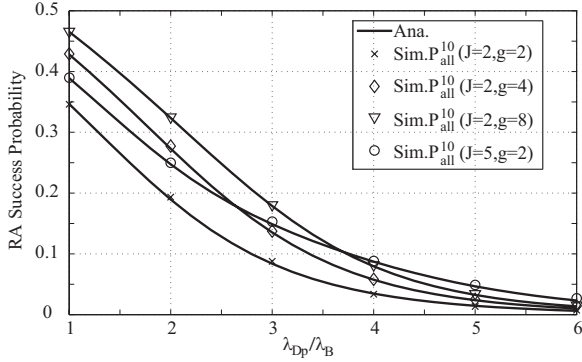


Fig. 4: RA success probability in the 10th time slot with the PR scheme.

VI. CONCLUSION

In this paper, we developed a spatio-temporal mathematical model to analyze the contention-based RA in the mMTC network by taking into account the SINR outage problem as well as the collision problem. We derived the exact expressions for the RA success probability in each time slot with the PR scheme. Numerical results shown that rapidly increasing transmit power is effective in increasing the RA success probability in a light traffic scenario, but become inefficiently when traffic improved.

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APPENDIX A A PROOF OF LEMMA 2

Let the typical IoT device transmit a preamble using the power level unit κ_1 , where the total number of IoT devices transmitting with the power level unit κ_1 is $N_1 + 1$ in this cell. Conditioning on this number N_1 , we first derive the PMF of N_2 number of IoT devices transmitting with power level unit κ_2 , which requires the Probability Density Function (PDF) of the area size of the Voronoi cell. Based on the Bayes' theorem [23, Eq. 2-44], the PDF of the area size of the Voronoi cell X conditioning on $N_1 = n_1$ is

$$\mathbb{P}[X = x | N_1 = n_1] = \frac{\mathbb{P}[N_1 = n_1 | X = x] \mathbb{P}[X = x]}{\mathbb{P}[N_1 = n_1]}. \quad (\text{A.1})$$

In (A.1), $\mathbb{P}[N_1 = n_1 | X = x]$ is the PMF of the number of interfering IoT devices N_1 in a cell conditioning on the area size of the cell $X = x$, presented as

$$\mathbb{P}[N_1 = n_1 | X = x] = \frac{(\mathcal{T}_{\kappa_1} \lambda_{Dp} x)^{n_1}}{\Gamma(n_1 + 1)} e^{-\mathcal{T}_{\kappa_1} \lambda_{Dp} x}, \quad (\text{A.2})$$

$$\begin{aligned}
\mathbb{P}[N_2 = n_2 | N_1 = n_1] &= \frac{(\mathcal{T}_{\kappa_2} \lambda_{Dp})^{n_2} (\mathcal{T}_{\kappa_1} \lambda_{Dp} + c \lambda_B)^{n_1+c+1}}{\Gamma(n_2+1) \Gamma(n_1+c+1)} \int_0^\infty x^{(n_2+n_1+c)} e^{-(\mathcal{T}_{\kappa_2} \lambda_{Dp} + \mathcal{T}_{PR, \kappa_1} \lambda_{Dp} + \lambda_B c)x} dx \\
&= \frac{(\mathcal{T}_{\kappa_2} \lambda_{Dp})^{n_2} (\mathcal{T}_{\kappa_1} \lambda_{Dp} + c \lambda_B)^{n_1+c+1}}{\Gamma(n_2+1) \Gamma(n_1+c+1)} \mathcal{L}_{x^{(n_2+n_1+c)}}(\mathcal{T}_{\kappa_2} \lambda_{Dp} + \mathcal{T}_{\kappa_1} \lambda_{Dp} + \lambda_B c) \\
&= \frac{(\mathcal{T}_{\kappa_2} \lambda_{Dp})^{n_2} (\mathcal{T}_{\kappa_1} \lambda_{Dp} + c \lambda_B)^{n_1+c+1} \Gamma(n_2+n_1+c+1)}{\Gamma(n_2+1) \Gamma(n_1+c+1) (\mathcal{T}_{\kappa_2} \lambda_{Dp} + \mathcal{T}_{\kappa_1} \lambda_{Dp} + \lambda_B c)^{n_2+n_1+c+1}}. \tag{A.7}
\end{aligned}$$

where \mathcal{T}_{κ_1} is the active probability of IoT devices transmitting with the power level unit κ_1 that will be derived in (14). $\mathbb{P}[X=x]$ is the PDF of the size of a cell that a randomly chosen IoT device belongs to, given in [11, Lemma 2]

$$\mathbb{P}[X=x] = \lambda_B \frac{c^{c+1}}{\Gamma(c+1)} (\lambda_B x)^c e^{-(\lambda_B c x)}, \tag{A.3}$$

and $\mathbb{P}[N_1 = n_1]$ is the PMF of N_1 number of interfering IoT devices transmitting with the power level unit κ_1 in the cell selected by the randomly chosen IoT device, given as [11, Eq.(3)]

$$\mathbb{P}\{N_1=n_1\} = \frac{c^{(c+1)} \Gamma(n+c+1) (\frac{\mathcal{T}_{\kappa_1} \lambda_{Dp}}{\lambda_B})^{n_1}}{\Gamma(c+1) \Gamma(n_1+1) (\frac{\mathcal{T}_{\kappa_1} \lambda_{Dp}}{\lambda_B} + c)^{n_1+c+1}}. \tag{A.4}$$

Substituting (A.2), (A.3), and (A.4) into (A.1), we obtain the PDF of the size of a cell conditioning on N_1 number of interfering IoT devices transmitting with the power level unit κ_1

$$\begin{aligned}
\mathbb{P}[X=x | N_1 = n_1] &= \\
&= \frac{(x)^{n_1+c} e^{-(\mathcal{T}_{\kappa_1} \lambda_{Dp} + \lambda_B c)x} (\mathcal{T}_{\kappa_1} \lambda_{Dp} + c \lambda_B)^{n_1+c+1}}{\Gamma(n_1+c+1)}. \tag{A.5}
\end{aligned}$$

Next, we derive the PMF of N_2 number of IoT devices transmitting with the power level unit κ_2 conditioning on the number of interfering IoT devices transmitting with the power level unit κ_1 in the typical Voronoi cell $N_1 = n_1$. Using the law of the total probability [23, Eq. 2-80], the PMF of N_2 number of IoT devices transmitting with the power level unit κ_2 in a Voronoi cell conditioning on $N_1 = n_1$ is expressed as

$$\begin{aligned}
\mathbb{P}[N_2 = n_2 | N_1 = n_1] &= \\
&= \int_0^\infty \mathbb{P}[N_2 = n_2 | X=x] \mathbb{P}[X=x | N_1 = n_1] dx. \tag{A.6}
\end{aligned}$$

Substituting (A.5) and (A.2) into (A.6), we obtain (A.7). Based on the iteration process, the PMF of N_j ($j = 3, 4, \dots, J$) number of active IoT devices transmitting with the power level units κ_j ($j = 3, 4, \dots, J$) in the Voronoi cell can be derived following (A.1) and (A.6), and we verified (6) in Lemma 2.

APPENDIX B A PROOF OF THEOREM 2

The preamble transmission success probability of an IoT device transmitting with the power level unit κ_j is represented

as

$$\begin{aligned}
\Theta(m, l, j, \vec{n}) &= \\
&= \exp\left(\frac{\gamma_{th}}{\kappa_j \rho} \sigma^2\right) \prod_{i=1}^J \left(\mathcal{L}_{\mathcal{I}_{inter_i}^m} \left(\frac{\gamma_{th}}{\kappa_j \rho} \right) \mathcal{L}_{\mathcal{I}_{intra_i}^m} \left(\frac{\gamma_{th}}{\kappa_j \rho} \middle| N_i = n_i \right) \right), \tag{B.1}
\end{aligned}$$

where $\mathcal{L}_{\mathcal{I}_{intra_i}^m}(\cdot)$ and $\mathcal{L}_{\mathcal{I}_{inter_i}^m}(\cdot)$ denote the Laplace Transform of the PDF of the aggregate intra-cell interference \mathcal{I}_{intra_i} and inter-cell interference \mathcal{I}_{inter_i} generating from the IoT devices transmitting with power level unit κ_i . The Laplace Transform of aggregate inter-cell interference from IoT devices transmitting with power level unit κ_i received at the typical BS is derived as

$$\begin{aligned}
\mathcal{L}_{\mathcal{I}_{inter_i}^m}(s) &\stackrel{(a)}{=} E_{\hat{\mathcal{Z}}_{out}} \left[\prod_{u_k \in \hat{\mathcal{Z}}_{out}} E_{P_k, h_k} [e^{-s \kappa_i P_k h_k \|u_k\|^{-\alpha}}] \right] \\
&\stackrel{(b)}{=} \exp \left(-2\pi \mathcal{T}_{PR, \kappa_i}^m \lambda_{Dp} \int_{(\frac{P}{\kappa_i \rho})^{\frac{1}{\alpha}}}^\infty E_{P, h} [1 - e^{-s \kappa_i P h x^{-\alpha}}] x dx \right) \\
&\stackrel{(c)}{=} \exp \left(-2\pi \mathcal{T}_{PR, \kappa_i}^m \lambda_{Dp} (\kappa_i s)^{\frac{2}{\alpha}} E_P [P^{\frac{2}{\alpha}}] \int_{(s \kappa_i \rho)^{\frac{1}{\alpha}}}^\infty \frac{y}{1 + y^\alpha} dy \right), \tag{B.2}
\end{aligned}$$

where $s = \frac{\gamma_{th}}{\kappa_j \rho}$, $E_x[\cdot]$ is the expectation with respect to the random variable x , $\mathcal{T}_{\kappa_i}^m$ is the active probability of IoT device transmitting with i th power level unit κ_i in the m th time slot, (a) follows from independence between λ_{Dp} , P_k , and h_k , (b) follows from the probability generation functional (PGFL) of the PPP, (c) obtained by changing the variables $y = \frac{x}{(sP)^{\frac{1}{\alpha}}}$, and the moments of the transmit power $E_P[\cdot]$ was presented in [12, Eq. A.2]. Substituting the moments of the transmit power into (B.2), we derive the Laplace Transform of aggregate inter-cell interference.

The Laplace Transform of aggregate intra-cell interference from IoT devices transmitting with the power level unit κ_i received at the typical BS is derived as

$$\mathcal{L}_{\mathcal{I}_{intra_i}^m}(s | N_i = n_i) = E_{h_k} \left[\exp \left(-s \sum_{k=1}^{n_i} \kappa_i \rho h_k \right) \right] = \left(\frac{1}{1 + s \kappa_i \rho} \right)^{n_i} \tag{B.3}$$

where n_i is the number of interfering IoT devices transmitting with the power level unit κ_i . Substituting (B.3) and (B.2) into (B.1), we obtain $\Theta(m, l, j, \vec{n})$.